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**Science and Technology Education**

**Document Series No. 20**

**Mathematics for All**

**Problems of cultural selectivity**

**and unequal distribution of mathematical education**

**and future perspectives**

**on mathematics teaching for the majority**

Report and papers presented in theme group I,

‘Mathematics for All’

at the 5th International Congress on Mathematical Education,

Adelaide, August 24-29, 1984

Edited by

Peter Damerow, Mervyn E. Dunkley, Bienvenido F. Nebres and Bevan Werry

**Division of Science**

**Technical and Environmental**

**Education**

UNESCO

ED-86/WS/84

**Preface**

This resource document consists of twenty-two papers prepared by authors from all regions and

presented at the Fifth International Congress on Mathematical Education (ICME 5). Over 2000

mathematics educators from sixty-nine countries gathered in Adelaide, Australia, in August 1984,

to discuss problems in their field. This document is one outcome. Its purpose is to continue the

dialogue to assist nations in their search for a mathematics programme for all students.

*Mathematics for All is* the first document in mathematics education in Unesco’s Science and

Technology Education Document Series. This, coupled with Unesco’s publications *Studies in*

*Mathematics Education* and *New Trends in Mathematics Teaching,* was initiated to encourage an

international exchange of ideas and information.

Unesco expresses its appreciation to the editors, Peter Damerow, Mervyn Dunkley, Bienvenido

Nebres and Bevan Werry for their work, to the Max Planck Institute for Human Development and

Education for preparing the manuscript, and to the ICME 5 Programme Committee for permitting

Unesco to produce this report.

The views expressed in the text are those of the authors and not necessarily those of Unesco,

the editors, or of ICME 5.

We welcome comments on the contents of this document. Please send them to: Mathematics

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Unesco, Place de Fontenoy, 75700 Paris, France.

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Introduction:

Report on the Work of Theme Group I

«Mathematics for All» at ICME 5

**1. Introduction**

Many factors have brought about a change in the overall

situation of mathematics education. These include the

move to universal elementary education in developing

countries, the move to universal secondary education in

industrialised countries (where there have also been growing

demands for mathematical competence in an increasingly

technologically and scientifically oriented world) and

from the experience gained with worldwide curriculum

developments such as the new mathematics movement.

The tacit assumption, that what can be gained from mathematics

can be gained equally in every culture and

independently of the character of the school institution and

the individual dispositions and the social situations of the

learner, turned out to be invalid. New and urgent questions

have been raised. Probably the most important ones are:

- What kind of mathematics curriculum is adequate to the

needs of the majority?

- What modifications to the curriculum or alternative curricula

are needed for special groups of learners?

- How should these curricula be structured?

- How could they be implemented?

A lot of work has already been done all over the world in

attempts to answer these questions or to contribute to special

aspects of the problem.

- ICME 4 yielded several presentations of results concerning

universal basic education, the relationship of

mathematics to its applications, the relation between

mathematics and language, women and mathematics,

and the problems of teaching mathematics to special

groups of students whose needs and whose situations

do not fit into the general framework of traditional

mathematics education.

-The Second International Mathematics Study of the

International Association for the Evaluation of

Educational Achievement (IEA) dealt much more than

the first one with the similarities and differences of the

mathematics curriculum in different countries, and the

different conditions which determine the overall outcome

in mathematical achievement. The IEA collected

data on both the selectivity of mathematics and the differences

between countries in the way they produce

yield levels of mathematical qualification. Although final

reports on the Second International Mathematics Study

are not yet available, preliminary analyses of the data

have already produced useful results.

- In several countries national studies have been concerned

with the evaluation of the mathematics education

system. An important recent example is the Report of

the Committee of Enquiry into the Teaching of

Mathematics in Schools in England and Wales (commonly

known as the Cockcroft Report) in 1982.

- Last, but not least, there are many detailed studies, projects

and proposals from different countries dealing with

special aspects such as:

- teaching the disadvantaged;

- teaching the talented;

- teaching mathematics to non-mathematicians;

- teaching mathematics in the context of real life situations;

- teaching mathematics under atypical conditions, etc.

At ICME 5, papers were presented on a variety of topics

related to the theme Mathematics for All. Taken as a whole,

these contribute to a better understanding of the problems

of teaching mathematics successfully, not only to very able

students, but teaching worthwhile mathematics successfully

to all in a range of diverse cultures and circumstances.

**2. Summary of Papers Presented to the Theme Group**

The first group of papers dealt with general aspects of the

theme Mathematics for All.

Jean-Claude Martin, Rector of the Academy of Bordeaux

in France, analysed in his paper, A *Necessary Renewal of*

*Mathematics Education,* the special selectivity of mathematical

education as a result of symbolism and mathematical

language. The teaching of mathematics seems to have

been designed to produce future mathematicians despite

the fact that only a very small percentage of students reach

university level. This general character of mathematical

education causes avoidable, system-related failures in

mathematical learning and often results in a strong aversion

to mathematics. Martin proposed a general reorientation

of mathematical education aiming at a mathematics

which is a useful tool for the majority of students. The teaching

of mathematics as a means of solving multidisciplinary

problems by using modelling methods should restore

student interest, show mathematics as being useful, enrich

students knowledge of related subjects and so enable

them better to memorise mathematical formulas and

methods, encourage logical reasoning and allow more students

access to a higher level of mathematics.

Bienvenido F. Nebres in his paper, *The Problem of*

*Universal Mathematics Education in Developing Countries,*

discussed the same problem of the lack of fit between the

goals of mathematical education and the needs of the majority

in the special circumstances of the situation in developing

countries. He offered a conceptual framework for discussing

the specific cultural dimensions of the problem in

these countries by using the distinction between vertical

and horizontal relationships, i.e. the relationships between

corresponding institutions in different societies and the rel

a t i o n ships between social or cultural institutions within the

same country. The history of social and cultural institutions

in developing countries is that their establishment and growth

has been guided more by vertical relationships, i.e. an

adaption of a similar type of institution from the mother colonial

country, rather than by horizontal relationships. The result

is a special lack of fit between mathematical education

and the needs of the majority of the people.

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There is a tremendous need for researchers in mathematics

education in developing countries to look at the

actual life of urban workers, rural farmers and merchants

and to identify the mathematics in daily life that is needed

and used by people. Then it is necessary to compare this

needed mathematics with what is provided in the curriculum

and to search for a better fit between the two. A cultural

shift must be brought about in these countries.

Mathematical educators, together with other educators and

other leaders of society, should take up the need for the

social and cultural institutions to be better integrated with

one another and to develop together in a more organic

manner than in the past.

In a joint paper *Mathematics for All: Conclusions Drawn*

*from the Experiences of the New Mathematics Movement,*

Peter Damerow of the Max Planck Institute for Human

Development and Education in Berlin, West Germany, and

Ian Westbury of the University of Illinois at

Urbana-Champaign, United States, examined the problem

of designing a mathematics curriculum which genuinely

meets the diverse needs of all students in a country. They

argue that, by continuing to ignore the needs of all except

a small minority of students, the curricula developed within

the new mathematics movement proved to be no more

satisfactory than their predecessors. Traditionally, mathematics

curricula were developed for an elite group of students

who were expected to specialise in the subject, and

to study mathematics subsequently at higher levels in a

tertiary institution. As education has become increasingly

universal, however, students of lesser ability, and with

more modest vocational aspirations and daily life requirements,

have entered the school system in greater numbers.

A major problem results when these students are

exposed to a curriculum designed for potential specialists.

This same type of traditional curriculum has frequently

been transferred to developing and third world countries,

where, because of different cultural and social conditions,

its inappropriateness for general mathematical education

has only been compounded. So called reforms such as

new mathematics did little to resolve the major problems in

that they merely attempted to replace one specialist curriculum

by another.

The question addressed by Damerow and Westbury is

how to cater both for the elite and also for the wider group

of students for whom mathematics should be grounded in

real world problem solving and daily life applications. One

suggestion is that the majority would achieve a mathematical

«Literacy» through the use of mathematics in other

subjects such as science, economics, while school mathematics

would remain essentially and deliberately for specialists.

This is effectively to retain the status quo.

Alternatively, mathematics must be kept as a fundamental

part of the school curriculum, but ways of teaching it effectively

to the majority must be found. The majority of students

will be users of mathematics. Damerow and

Westbury concluded that a mathematics program which is

truly for all must seek to overcome the subordination of elementary

mathematics to higher mathematics, to overcome

its preliminary, preparatory character, and to overcome its

irrelevance to real life situations.

The findings of the Second International Mathematics

Study (SIMS) were used by Howard Russell, Ontario

Institute for Studies in Education, Canada, in his paper

*Mathematics for All: SIMS Data,* to argue that mathematics

is already taught to all pupils at the elementary level in

many countries. At the senior secondary level, however,

the prevailing pattern in most countries is for mathematics

to be taught only to an elite. At the lower level, the SIMS

data suggest that promotion by age, rather than by performance,

does not violate the concept of mathematics for all.

The SIMS data also appear to provide support for the

Cockcroft hypothesis that the pace of mathematics education

must be slowed if sufficient students are to be retained

in mathematics courses at the higher levels for it to be

accurately labelled mathematics for all. Alternatively, the

content of the curriculum could be trimmed down as suggested

by Damerow. Russell proposed a market-oriented

rationale to construct such a core of material, particularly to

meet the needs of the middle level students who will be

required to use mathematics in their chosen work in the

market place.

Afzal Ahmed was a member of the Committee of Inquiry

into the Teaching of Mathematics in Schools in England

and Wales (Cockcroft Committee), and is now the director

of the Curriculum Development Project for Low Attaining

Pupils in Secondary School Mathematics. In his paper, *The*

*Foundations of Mathematics Education for All,* he discussed

implications of the Cockcroft Report, published in January

1982, concerning the major issues of the theme group. He

pointed out that a suitable mathematics curriculum for the

majority assumes greater importance as societies in the

world become more technological and sophisticated. But at

the same time, the evidence of failure at learning and

applying mathematics by a large proportion of the population

is also growing. The Cockcroft Report proposes a

Foundation List of Mathematical Topics that should form

part of the mathematics syllabus for all pupils. In his discussion

of the Cockcroft Report, Ahmed focussed

particularly on the classroom conditions which facilitate, or

inhibit the mastery of these fundamental topics.

In a paper on *Universal Mathematics Education,*

Achmad Arifin from the Bandung Institute of Te c h n o l o g y

in Indonesia, described how community participation

should be raised in carrying out universal mathematics

education through looking at the aspect of interaction

within and between social and cultural institutions. He

asked the questions which parts of mathematics can

function as a developer of an individual’s intelligence and

how should those parts that have been chosen be presented?

Any program to answer these questions has to

take into account three components of interaction. Firstly,

depending on its quality, social structure through interaction

can contribute to the improvement of peoples’ a b i l ities,

especially by making them appreciate mathematics.

S e c o n d l y, a special form of social interaction, which he

called positive interaction, can motivate mathematics

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learning and create opportunities to learn. Thirdly, school

interaction itself can inspire, stimulate, and direct learning

activities. In developing countries, local mathematicians in

particular are able to understand their cultural conditions,

the needs, the challenges and the wishes of their developing

nation. Taking into account the three components of

interaction, they have the ability and the opportunity to

spread and share their knowledge and to translate and utilise

the development of mathematics in universal

mathematics education for their nation.

In many countries, there is one mathematics syllabus for

each year of the education system. Andrew J. C. Begg, in

his paper, *Alternative Mathematics Programs,* questioned

this practice and argued for the introduction of alternative

mathematics programs which will meet the varied needs of

all students in a range of circumstances and with a range

of individual aspirations. All such courses should contribute

towards general educational aims such as the development

of self-respect, concern for others, and the urge to

enquire. Thus, mathematics courses should provide an

opportunity to develop skills of communication, responsibility,

criticism, and cooperation. Such an approach has

implications for the way in which students are organised in

mathematics classes; for the scheduling of mathematics

classes; for the choice of teaching and learning methods;

for the extent to which emphasis is placed on cooperation

as against competition; for the use of group methods of

teaching; and for the provision that should be made for students

from diverse cultural groups. In this way, mathematics

programs for all students should assist not only the

achievement of mathematical objectives, but also the

attainment of personal, vocational and humanistic aims in

education. By matching mathematics programs to the

needs of students, the development of the self-esteem of

every student becomes central in the mathematics curriculum.

The second group of papers was concerned with particular

concerns related to Mathematics for All in industrialised

countries.

In their paper, *Arithmetic Pedagogy at the Beginning of the*

*School System in Japan,* Genichi Matsubara and

Zennosuke Kusumoto traced the introduction of the teaching

of western arithmetic to Japan in the late nineteenth

century. At a time when universal elementary education

was only just approaching reality in Japan, the government

declared a policy of adopting western-style arithmetic in

order to enable the country to compete more successfully

internationally. This move faced obstacles in its implementation

because of the traditional use of the abacus and the

widespread lack of familiarity with the Hindu-Arabic notation.

Further, in a developing national system of education,

teachers were in short supply and little attention could be

given to teaching methods in the training courses. The

paper emphasised the need to make such changes slowly

and to take into account the situation of those closely involved

with the changes if they are to be successful in modifying

the curriculum for mathematics for all.

The extent to which the mathematics learnt at school is

retained and used in later life is the subject of research

reported in a paper by Takashi Izushi and Akira Yamashita

of Fukuoka University, Japan, entitled *On the Value of*

*Mathematical Education Retained by the Social Members of*

*Japan in General.* A study in 1955 was concerned with

people who had learnt their mathematics before the period

in Japan in which mathematics teaching was focussed on

daily life experience and before compulsory education was

extended to secondary schools. Although it was found that

most people retained the mathematics skills and knowledge

well, rather fewer claimed that this material was useful

in their work. A second more limited study in 1982 confirmed

these general findings in relation to geometry. It showed,

broadly speaking, that younger people tended to use

their school mathematics more directly while older people

relied more on common sense. The study covered a further

aspect, the application of the attitudes of deductive thinking

derived from the learning of geometry. The thinking and

reasoning powers inculcated by this approach were not forgotten

and were claimed to be useful in daily life, but not in

work. Izushi and Yamashita conclude that the inclusion of

an element of formal mathematical discipline in the curriculum

is supported by Japanese society.

Another attempt to create a modern course in advanced

mathematics which is also worthwhile for those students

who don’t intend to proceed to university was reported by

Ulla Kürstein Jensen from Denmark in her paper titled

*Upper Secondary Mathematics for All? An Evolution and a*

*Draft.* The increase from about 5% in former years to about

40So in 1983 of an age cohort completing upper secondary

education with at least some mathematics brought about

an evolution toward a curriculum concentrating on useful

mathematics and applications in daily life and mathematical

modelling. This evolution led to the draft of a new curriculum

which will be tested under school conditions, beginning

in autumn 1984. The origin of this development is

based on new regulations for mathematical education for

the upper secondary school in the year 1961. It was

influenced by the new mathematics ideas and designed to

serve the needs of the small proportion of the students

passing through upper secondary education at that time,

but soon had to be modified for the rapidly increasing number

of students in the following years. So the mathematics

teaching, particularly for students in the language stream of

the school system, was more and more influenced by ideas

and teaching materials of a further education program

which was much more related to usefulness for a broad

part of the population than the usual upper secondary

mathematics courses. In 1981, this development was legitimated

by new regulations and, by that time, even mathematics

teaching in classes concentrating on mathematics

and physics became more and more influenced by the tendency

to put more emphasis on applications leading ultimately

to the draft of the new unified curriculum which is

now going to be put into practice.

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The central topic of a paper entitled *Fight against School*

*Failure in Mathematics,* presented by Josette Adda from the

Université Paris 7, was an analysis of social selective functions

of mathematical education. She reported statistical

data showing the successive elimination of pupils from the

«normal way at each decision stage of the school system

until only 16% of the 17 year age cohort remain whereas

all others have been put backward or relegated to special

types of classes. These eliminations hit selectively

socioculturally disadvantaged families. Research studies,

particularly at the Université Paris 7, have been undertaken

to find out why mathematics teaching as it is practised

today is not neutral but produces a correlation between

school failure in mathematics and the sociocultural environment.

They indicate the existence of parasitic sources

of misunderstanding increasing the difficulties inherent in

mathematics, e. g. embodiments of mathematics in pseudo-

concrete situations which are difficult to understand for

many pupils. On the other hand, it had been found that children

failing at school are nevertheless able to perform

authentic mathematical activities and to master logical operations

on abstract objects.

Two papers were based on the work of the EQUALS

program in the United States. This is an intervention program

developed in response to a concern about the high

dropout rate from mathematics courses, particularly in the

case of women and minority students. The program aims

to develop students’ awareness of the importance of

mathematics to their future work, to increase their confidence

and competence in doing mathematics, and to

encourage their persistence in mathematics.

In the first of these papers, *EQUALS: An Inservice*

*Program to Promote the Participation of Underrepresented*

*Students in Mathematics,* Sherry Fraser described the way

in which the program has assisted teachers to become

more aware of the problem and the likely consequences

for individual students of cutting themselves off from a

mathematical education. By working with teachers and

providing them with learning materials and methods, with

strategies for problem solving in a range of mathematical

topics, together with the competence and confidence to

use these, EQUALS has facilitated and encouraged a

transfer of concern to the classroom and attracted and

retained greater numbers of underrepresented students in

mathematics classes. Since 1977, 10,000 educators have

participated in the program.

Although the main focus of activity in the EQUALS program

has been on working with teachers and administrators,

needs expressed by these educators for

materials to involve parents in their children’s mathematical

education led to the establishment of *Family Math.* Virginia

Thompson described how this project has developed a curriculum

for short courses where parents and their children

can meet weekly to learn mathematical activities together

to do at home. This work reinforces and complements the

school mathematics program. Although the activities are

suitable for all students, a major focus has been to ensure

that underrepresented students, primarily females and

minorities, are helped to increase their enjoyment of

mathematics. The project serves to reinforce the aims of

the EQUALS program.

The move over the past ten years or so towards applicable,

real world and daily life mathematics in the

Netherlands, inspired by the work of Freudenthal, was described

by Jan de Lange Jzn. of OW and OC, Utrecht, in his

*paper Mathematics for All is No Mathematics at All.* Textbooks

have been published for primary and lower secondary

schools which reflect this view of mathematics, and research

shows that the reaction of teachers and students has

been very favourable. De Lange illustrated the vital role

played by applications and modelling in a newly-introduced

curriculum for pre-university students. Many teachers

apparently view the applications-oriented approach to

mathematics very differently from the traditional mathematics

content. The ultimate outcome, de Lange suggested,

may be that science and general subjects will absorb the

daily life use of mathematics and consequently this type of

mathematics might disappear from the mathematics curriculum.

That is, the ultimate for all students as far as mathematics

is concerned could in reality become no mathematics

as such.

Roland Stowasser from the Technical University of West

Berlin proposed in his paper, *Problem Oriented Mathematics*

*Can be Taught to All,* to use examples from the history of

mathematics to overcome certain difficulties arising from

courses based on a single closed system, which increase

mathematical complexity but do not equally increase the

applicability to open problems. He stated that mathematics

for all does not necessarily have to be directly useful, but it

has to meet two criteria: The mathematical ideas have to

be simple, and on the other hand, they have to be powerful.

He illustrated these criteria through a historical

example. Regiomantus formulated the problem to find the

point from which a walking person sees a given length high

up above him (e. g. the minute hand of a clock if the person

walks in the same plane as the face of the clock) subtending

the largest possible angle. The solution with ruler

and compasses in the framework of Euclidean geometry is

somewhat tricky. But according to Stowasser the teaching

of elementary geometry should not be restricted to Greek

tricks. For problem solving he advocated free use of possible

tools, and the solution of the problem is very simple if

trial and error methods are allowed. So the solution of the

historical problem represents the simple but powerful idea

of approximation.

What are the characteristics of a mathematics program

suitable for all students, and do any such programs exist?

These questions were addressed by Allan Podbelsek of

the United States in his paper, *Realization of a Mathematics*

*Program for All.* Podbelsek listed a number of criteria for

such a program covering not only content knowledge and

skills but also attitudes towards, and beliefs about, mathematics

and the process skills involved in its use. Mathematics

must be seen to be a unified, integrated subject, rather

than a set of individual, isolated topics. T h e

Comprehensive School Mathematics Program

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(CSMP) developed over several years in the United States

for elementary (K-6) level classes is found to meet these

criteria successfully in almost every respect. Practical problems

involved in the introduction of such a program as

CSMP to a school were discussed by Podbelsek. These

problems centred on the provision of adequate teacher

training for those concerned, meeting the cost of materials,

securing the support of parents and the local community,

and ensuring that administrative staff were aware of the

goals of the program.

Those translating mathematical, scientific or technical

material should have a basic knowledge of mathematics to

do their job satisfactorily, yet because of their language

background they are not likely to have studied mathematics

to any great extent at school. This is the experience

which led Manfred Klika, of the Hochschule Hildesheim in

West Germany, to a consideration of the nature and adequacy

of present school mathematics programs in his

paper *Mathematics for Translators Specialised in Scientific*

*Texts - A Case Study on Teaching Mathematics to*

*Non-Mathematicians.* Conventional school programs, he

claimed, do not prepare students to comprehend and make

sense of mathematical ideas and terminology. The solution

is to construct the mathematics curriculum around fundamental

ideas. Two perspectives on this notion are offeredmajor

anathematising models (e. g. mathematical

concepts, principles, techniques, etc.) and field-specified

strategies suitable for problem solving in mathematics (e.g.

approximate methods, simulation, transformation strategies,

etc.). A curriculum based on such fundamental ideas

would result in more meaningful learning and thus a more

positive attitude to the subject. A course based on this

approach has been established at the Hochschule

Hildesheim within the program for training specialist translators

for work in technical fields.

The major concern of the preceding contributions to the

topic “ Mathematics for All ” were problems of designing a

mathematics curriculum which is adequate to the needs

and the cognitive background of the majority in industrialised

countries. The organising committee of the theme

group was convinced that it is even more important to discuss

the corresponding problems in developing countries.

But it was much more difficult to get substantial contributions

in this domain. To stress the importance of the development

of mathematical education in developing countries,

the work of the theme group terminated with a panel discussion

on *Universal Mathematical Education in Developing*

*Countries,* with short statements of major arguments by

Bienvenido F. Nebres from the Philippines, Terezinha N.

Carraher from Brazil, and Achmad Arifin from Indonesia,

followed by the reactions of Peter Towns and Bill Barton,

both from New Zealand. The discussion concentrated on

the relation between micro-systems of mathematical education

like curricula, textbooks and teacher training and

macro-systems like economy, culture, language and general

educational systems which, particularly in the developing

countries, often determine what kind of developments

on the level of micro-systems are possible. Bienvenido F.

Nebres expressed the common conviction of the participants

when he argued that, in spite of the fact that often it

is impossible to get a substantial improvement of

mathematics education without fundamental changes in

the macro-systems of education, micro-changes are possible

and are indeed a necessary condition to make people

realise what has to be done to get a better fit between

mathematical education and the needs of the majority. This

result of the discussion highlights the importance of the

papers submitted to the theme group dealing with special

aspects of mathematical education in developing countries.

Three reports were given by David W. Carraher,

Terezinha N. Carraher and Analucia D. Schliemann about

research undertaken at the Universidade Federal de

Pernambuco in Recife, Brazil. David W. Carraher prepared

a paper titled *Having a Feel for Calculations* about a study

investigating the uses of mathematics by young, schooled

street vendors who belong to social classes characteristically

failing in grade school, often because of problems in

mathematics, but who often use mathematics in their jobs

in the informal sector of the economy. In this study, the

quality of mathematical performance was compared in the

natural setting of performing calculations in the market

place and in a formal setting similar to the situation in a

classroom. Similar or formally identical problems appeared

to be mastered significantly better in the natural setting.

The reasons were discussed and it was stated that the

results of the analysis strongly suggest that the errors

which the street vendors make in the formal setting do not

reflect a lack of understanding of arithmetical operations

but rather a failing of the educational system which is out

of touch with the cognitive background of its clientele.

There seems to be a gulf between the intuitive understanding

which the vendors display in the natural setting and

the understanding which educators try to impart or develop.

Terezinha N. Carraher reported in her paper *C a n*

*Mathematics Teachers Teach Proportions?* results of a

second research project. Problems involving proportionality

were presented to 300 pupils attending school in

Recife, in order to find out whether a child already understands

proportions if it only follows correctly the routines

being taught at school. The results indicate characteristic

types of difficulties appearing in certain problems, some of

which can be related to cognitive development. It is suggested

that teachers’ awareness of such difficulties may

help to improve their teaching of the subject. For if mathematics

is to be useful to everyone, mathematics teachers

must consider carefully issues related to the transfer of

knowledge acquired in the classroom to other problem solving

situations.

The third paper, presented by Analucia D. Schliemann,

*Mathematics Among Carpentry Apprentices: Implications for School*

*Teaching,* highlighted the discontinuity between formal school

methods of problem solving in mathematics and the informal

methods used in daily life. This research study contrasted the

approaches to a practical problem of quantity estima-

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tion and associated calculation taken by a group of experienced

professional carpenters without extensive schooling,

and a group of carpentry apprentices attending a formal

school system and with at least four years of mathematics

study. The results showed that apprentices approached

the task as a school assignment, that their strategies

were frequently meaningless and their answers absurd. On

the other hand, the professional carpenters took it as a

practical assignment and sought a feasible, realistic solution.

Very few computational mistakes were made by either

group but the apprentices appeared unable to use their formal

knowledge to solve a practical problem. Schliemann

concluded that problem solving should be taught in practical

contexts if it is to have transferability to daily life situations

out of school.

Pam Harris from the Warlpiri Bilingual School discussed

in her paper, *Is Primary Mathematics Relevant to Tribal*

*Aboriginal Communities?,* the problem that, in the remote

Aboriginal communities of Australia, teachers often get the

feeling that mathematics is not relevant. Several reasons

can be identified. Teachers often receive negative attitudes

from other people so that they go to an Aboriginal community

expecting that their pupils will not be able to do mathematics.

Furthermore, they observe a lack of reinforcement

of mathematics in the pupils’ home life. Teaching materials

mostly are culturally and linguistically biased. Teachers feel

discouraged because of the difficulties of teaching mathematics

under these conditions. Nevertheless, Pam Harris

stressed the importance of mathematics, because

Aboriginal children have to get an understanding of the

Second culture» of their country. They need mathematics

in their everyday life, in employment, and in the conduct of

community affairs. But to be successful, mathematics

teaching in Aboriginal communities has to allow for and

support local curriculum development. Individual schools

and language groups should make their own decisions on

the use of the children’s own language, the inclusion of

indigenous mathematical ideas, priorities of topics, and

sequencing the topics to be taught.

Kathryn Crawford, from the College of A d v a n c e d

Education in Canberra, presented a paper on *Bicultural*

*Teacher Training in Mathematics lEducation for Aboriginal*

*Trainees from Traditional Communities* in Central Australia.

She described a course which forms part of the Anagu

Teacher Education Program, an accredited teacher training

course intended for traditionally oriented Aboriginal

people currently residing in the Anagu communities who

wish to take on greater teaching responsibilities in South

Australian Anagu schools. The most important difference

between this teacher training course and many others is

that it will be carried out on site by a lecturer residing within

the communities and that, from the beginning, development

of the curriculum has been a cooperative venture between

lecturers and educators on the one hand, and community

leaders and prospective students on the other. The

first group of students will begin the course in August 1984.

The course is particularly designed to meet the fact that different

cultures emphasise different conceptual schemes.

Thus, temporal sequences and quantitative measurement

are dominant themes in industrialised Western cultures but

largely irrelevant in traditional Aboriginal cultures. To overcome

these difficulties, the focus of the problem is redirected

from the “failings” of Aboriginals and Aboriginal

culture to the inappropriateness of many teaching practices

for children from traditionally oriented communities.

The course has been developed based on a model designed

to maximise the possibility of interaction between the

world view expressed by Anagu culture and that of

Anglo-European culture as evidenced in school mathematics.

This is achieved by placing an emphasis on the student

expertise and contribution in providing information

about Anagu world views as a necessary part of the course.

In this community based teacher training course, it

seems that it is possible for the first time to develop procedures

for negotiating meanings between the two cultures.

**3. Conclusions**

The presentations given at the sessions of the theme

group summarised above can be considered as important

efforts to contribute to the great program of teaching

mathematics successfully not only to a minority of selected

students but teaching it successfully to all. But in spite of all

these efforts it has to be admitted that the answer to. the

question, What kind of mathematics curriculum is adequate

to the needs of the majority?», is still an essentially open

one. However, the great variety of the issues connected

with this problem which were raised in the presented

papers makes it at least clear that there will be no simple

answer. Thus the most important results of the work of this

theme group at ICME 5 may be that the problem was for

the first time a central topic of an international congress on

mathematical education, and that, as the contributions

undoubtedly made clear, this problem will be one of the

main problems of mathematical education in the next decade.

As far as the content of these contributions is concerned,

the conclusion can be drawn that there are at least three

very different dimensions to the problem which contribute

to and affect the complex difficulties of teaching mathematics

effectively to the majority:

- the influence of social and cultural conditions;

- the influence of the organisational structure of the school

system;

- the influence of classroom practice and classroom

interaction.

*Cultural Selectivity*

One of the major underlying causes of the above problem

is the fact that mathematical education in the traditional

sense had its origins in a specific western European cultural

tradition. The canonical curriculum of «Tr a d i t i o n a l

mathematics» was created in the 19th century as a study

for an elite group. It was

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created under the conditions of a system of universal basic

education which included the teaching of elementary computational

skills and the ability to use these skills in daily life

situations. There is a clear distinction between the aims

and objectives of this basic education and the curriculum of

traditional school mathematics which was aimed at formal

education not primarily directed at usefulness and relevance

for application and practice. This special character of

the canonical mathematics school curriculum is still essentially

the same today in many countries.

The transfer of the European mathematics curriculum to

developing countries was closely associated with the establishment

of schools for the elite by colonial administrations.

Under these circumstances it seemed natural to simply

copy European patterns. It is quite another problem to

build a system of mass education in the Third World and

embed mathematics education in both the school situation

and the specific social and cultural contexts of that world.

The papers summarised above point clearly to some of

the problems. Curricula exist which encourage students to

develop antipathies towards mathematics; this is commonly

the case in Europe. Further, such curricula have sometimes

been transferred to countries where the social

context lacks the culturally based consensus that is found

in Europe, namely, that abstract mathematical activity is

good in itself and must therefore be supported, even if it

seems on the surface to be useless. It has been proposed

on the one hand, that a sharp distinction should be made

between applicable arithmetic in basic education and essentially

pure mathematics in secondary education, and on

the other hand, that mathematics should be integrated into

basic technical education. This argument raises the question

of the relation between mathematics and culture which

may be the first problem to address when the idea of

mathematics for all is raised as a basis for a program of

action.

*Selectivity of the School System*

While the particular curricular patterns of different societies

vary, the subject is still constructed in most places so that

few of the students who begin the study of mathematics

continue taking the subject in their last secondary years.

The separation of students into groups who are tagged as

mathematically able and not able is endemic. Curricula are

constructed from above, starting with senior levels, and

adjusted downwards. The heart of mathematics teaching

is, moreover, widely seen as being centered on this curriculum

for the able, and this pattern is closely related to

the cultural contexts indicated above. However, we must

consider the problem of conceiving, even for industrialised

societies, a mathematics which is appropriate for those

who will not have contact with pure mathematics after their

school days. Up to now we have made most of our students

sit at a table without serving them dinner. Most

attempts to face the problem of a basic curriculum reduce

the traditional curriculum by watering down every

mathematical idea and every possible difficulty to make it

feasible to teach the remaining skeleton to the majority.

There is only a limited appeal to usefulness as an argument

or a rationale for curriculum building to avoid the

pitfalls of this situation. Students who will not have to deal

with an explicit area of pure mathematics in their adult lives

but will face instead only the exploitation of the developed

products of mathematical thinking (e. g. program packages),

will only be enabled by mathematics instruction if

they can translate the mathematical knowledge they have

acquired into the terms of real-life situations which are only

implicitly structured mathematically. Very little explicit

mathematics is required in such situations and it is possible

to survive in most situations without any substantial mathematical

attainments whatsoever.

Is the only alternative to offer mathematics to a few as a

subject of early specialisation and reject it as a substantial

part of the core curriculum of general education? This

approach would deny the significance of mathematics. To

draw this kind of conclusion we would be seen to be looking

backwards in order to determine educational aims for

the future. The ongoing relevance of mathematics suggests

that a program of mathematics for all implies the

need for a higher level of attainment than has been typically

produced under the conditions of traditional school

mathematics — and that this is especially true for mathematics

education at the level of general education. In other

words, we might claim that mathematics for all has to be

considered as a program to overcome the subordination of

elementary mathematics to higher mathematics, to overcome

its preliminary character, and to overcome its irrelevance

to life situations.

*Selectivity in Classroom Interaction*

Some of the papers presented in this theme group support

recent research studies which have suggested that it

is very likely that the structure of classroom interaction

creates ability differences among students which grow

during the years of schooling. In searching for causes of

increasing differences in mathematical aptitude, perhaps

the simplest explanation rests on the assumption that

such differences are due to predispositions to mathematical

thinking, with the further implication that nothing can

be done really to change the situation. But this explanation

is too simple to be the whole truth. The understanding

of elementary mathematics in the first years of primary

school is based on preconditions such as the acquisition

of notions of conversation of quantity which are, in their

turn, embedded in exploratory activity outside the school.

The genesis of general mathematical abilities is still little

understood. The possibility that extra-school experience

with mathematical or pre-mathematical ideas influences

school learning cannot be excluded. Furthermore, papers

presented to the theme group strongly suggest that the

d i fferences between intended mathematical understanding

and the understanding which is embedded in normal

classroom work is vast. We cannot exclude the possibility

that classroom interaction in fact produces growing differences

in mathematical aptitude and achievement by a

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system of positive feedback mechanisms which increase

high achievement and further decrease low achievement.

It is clear that to talk of mathematics for all entails an

intention to change general attitudes towards mathematics

as a subject, to eliminate divisions between those who are

motivated towards mathematics and those who are not,

and to diminish variance in the achievement outcomes of

mathematics teaching. This, in its turn, involves us in an

analysis of social contexts, curricula and teaching. It is

these forces together which create a web of pressures

which, in turn, create situations where mathematics

becomes one of the subjects in the secondary school in

which selection of students into aptitude and ability groups

is an omnipresent reality almost from the time of entry.

**Part I:**

**Mathematics for All –**

**General Perspectives**

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**A Necessary Renewal of Mathematics**

**Education**

Jean-Claude Martin

Mathematics for all must not only be accessible mathematics,

but interesting mathematics for all - or for the majority.

Such a theory leads one, in the case of the teaching of

mathematics in France, to raise problems of objectives and

curriculum organisation, but also of methods more than of

the content of the curriculum.

**1. The General Characteristics of Selective Education**

*(i) The Fundamental Teaching of Mathematics*

*for Mathematics Sake*

Mathematics as they are known today may be considered,

if not as a whole, as a system. The training of the highest

level of generalist mathematicians may a priori be defined

as leading to knowledge of this system.

Dividing the system of mathematics into parts going

from the simplest element to the most complicated may

represent, as a first approximation only, but quite logically,

a curriculum of study for the training of mathematicians.

That is what we shall call, to serve as a reference for our

later discussions, the teaching of mathematics for mathematics

sake. Its organisation in the form of a continuous

upward progression implies that each level reached will be

a prerequisite for the level immediately following.

Such a curriculum does not exist in the pure state but it

appears to be the foundation, the skeleton of most programs

of general mathematical training in many countries,

being a reflection of the European rationalist cultural tradition.

Adaptations of this consist essentially in heavier or lighter

pruning, stretching to varying degrees the progression,

or illustrating it to some extent by an appeal to real-life

experience (either in order to introduce a notion or to

demonstrate some application of it).

The first question raised then is whether such teaching

is a suitable basis for mathematics for all.

On the level of objectives, the reply is obviously negative:

The training of mathematicians can interest only a

minute portion of students.

*(ii) Selection by Means of Mathematics*

In France, statistics show that of any 1,000 students entering

secondary education, fewer than 100 will obtain seven

years later a scientific baccalaureat (including section D)

and a maximum of five will complete tertiary studies in

mathematics or related disciplines (computer science in

particular).

Referring again to statistics indicates that only about

one successful candidate at the baccalaureat in six holds

one of the types of baccalaureat (C or E) in which mathematics

are preponderant. That fact, together with other

indications concerning class counselling, brings out sufficiently

clearly the importance of selection — a well enough

known phenomenon anyway — by mathematics in the

secondary school. This selectivity appears moreover to be

relatively stronger than at university. This situation makes

mathematics a dominant subject. French, which formerly

shared the essential role in selection, is now relegated to a

secondary position.

This selection is manifested most often by a process of

orientation through failure for students at certain levels. But

in fact, this sanction is usually only the deferred result of an

ongoing selection process which takes effect cumulatively.

From the primary school, or as early as the first years of

secondary school, the classification between «Maths» and

«non-maths» students becomes inexorably stratified.

In recent years, the idea that selection through mathematics

is equivalent to a selection of intelligent students

has made some progress, even if it is only very rarely

expressed in such a clear way.

This function as the principal filter of the education system

has considerably harmed the prime constructive function

of mathematics as a means of training thought processes

by the practice of logical reasoning. Just as a filter

naturally catches waste, so mathematics produce academic

failures inherent in the system, in other words, not due

to intrinsically biological or psycho-effective causes but to

the teaching process itself.

The type of evaluation used is not unconnected. It has

the general fault of all standardised evaluation as is still too

widely practised.

It supposes a definition of the child’s normality that

pediatricians and psychologists contests1,2 : ranges of

development, differences in maturity are just as normal

and natural as differences in height and body weight. The

same applies to the formation and development of abstract

thought, which one must expect to be facilitated by the teaching

process and not measured and sanctioned by it.

Aptitude for abstraction seems to be generally considered,

with intelligence, as having an essentially innate character,

whereas it is admitted by researchers that the share

acquired in the social and family milieu and then at school

is probably preponderant.

The demands of «Levels of intelligence» are also judged

excessive for the teaching situation (first years of

secondary school). They would necessitate2 a clearly

above average IQ.

On this subject we may note a very important lack of

coordination between the quite reasonable programs and

instruction of the Inspectorate General of Teaching and the

contents of textbooks.

We shall see later some questions concerning the vocabulary

used, but where the program considers only arithmetic

or operations on whole numbers or rational numbers,

it can be seen that in fact a veritable introduction to algebra

is carried out.

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*(iii) Emotional Responses and Mathematics*

All teaching is obviously subject to emotional responses:

the student likes this, doesn’t like that, prefers this, and so

on. As far as mathematics is concerned, successful students

acquire an assessment based on a harmonious relationship,

but those who have difficulties feel strong emotions

that can induce suffering and anguish.4

**II. Problems of Language, Symbolic Writing**

A mathematical apprenticeship requires the acquisition of a

special language which is characterised by the interlocking

of a conventional language (with nevertheless its own

semantics and syntax) and a symbolic language.

If, beside their communicative aim, all languages serve

as a medium of thought — according to Sapir: The feeling

that one could think and indeed reason without language is

an illusions — the language of mathematics, more than any

other, is adapted to that very end. The sentence (containing

words) and the formula with its symbols are vehicles

of logical reasoning. In this area, symbolic writing is considerably

more powerful than conventional writing: one can

say that it is a motive force driving thought ahead more

rapidly.

*(i) The Power of Symbols*

On the occasion of the 4th International Congress on the

Teaching of Mathematics (ICME 4), Howsons5 clearly showed

the power of symbols, which one could have thought

in the first analysis to be only tricks of abbreviation, whereas

they do generate new meanings.

As essential elements of mathematics, they permit the

discipline to develop without its being necessary to burden

our thought processes with all the meanings with which

they are charged. A language open to independent development,

symbolic writing lends itself to operations the

automatic nature of which, once it is acquired, saves

conscious thought or at the very least permits considerable

economies in the process of reflexion.

An apprenticeship in symbolic writing and the attendant

operational procedures is therefore essential in the teaching

of mathematics.

*(ii) The Importance of Language Acquisition*

The nature of symbolic writing being a capacity for

self-development, if what has been learned in this area is

already considerable, the student will have no major difficulty

in acquiring the language necessary if he is to pass to

the next stage. Thus his difficulties will reside rather in the

structures of reasoning than in a knowledge of symbols. It

may be considered that this is the case of students in the

upper classes of secondary school.

On the other hand, at the beginning of this apprenticeship

(notably when algebra is introduced), the change

from the natural language to symbolic language, because

it is a prerequisite, no doubt has a special place in the hierarchy

of difficulties.

*(Iii) The Difficult Changeover to Symbolism*

The changeover from natural language to symbolic language,

as well as the problems caused by too rapid or too

early an introduction (poorly adapted to the development of

the thought processes of the student and his maturity) carries

with it some more technical difficulties, which in our

view have not been satisfactorily solved.

Symbolic formulation is more than mere translation. The

physicist is well aware of this, considering as he does

today this operation, called (mathematical) modelling, as

being of prime importance in the analysis of complex phenomena

or systems. In the same way, the return from the

formula to realist is an exercise that is not self-evident and

a table of correspondences and a dictionary will not suffice.

Symbolism introduces first of all a complication.

Afterward, naturally, when the obstacle is overcome, one

profits as a result of a simplification of procedures (automatic

responses in operations and their reproduction).

If one can solve a problem in ordinary language with a

level of difficulty N1, to use for its solution a poor knowledge

of symbolic language makes it more difficult (level N2).

On this subject the tests of C. Laborde7 seem significant.

Confronted with solving concrete problems or describing

mathematical objects, students do not use the codes they

have learned. But once the symbolism is better known, the

level of effort to attain the same goal is less. Level N3 is for

example the level of effort required of the master mathematician.

This summary demonstrates at the same time the

advantage of learning mathematics and the difficulty there

is, starting with the concrete description of a problem to formulate

it in mathematical terms. It also shows that the teacher

should give considerably more attention to lessening

the difficulty of acquiring the mathematical metalanguage

than accumulating purely mathematical knowledge.

*(iv) The Necessity of Introducing Stages Useful for*

*Conceptualisation*

G. Vergnaud8 has demonstrated that, when solving problems,

students used Faction theorems» or implicit theorems,

which were simply the products of their personal

conceptualisation revealing the workings of individual

thought processes. Several researchers have noted that

such processes did not follow the shortest path of the

mathematics taught nor the best method from the point of

view of logical rigour.

Because of this, it is often considered by the teacher to

be bad reasoning - to be done away with as quickly as possible

in favour of classical mathematical reasoning — whereas

it is rather logical reasoning in the process of developing.

The act of teaching, instead of ignoring or indeed rejecting

the representation constructed by the pupil, his own

personal mechanisms of thought, should consist, on the

contrary, in revealing these, understanding them and using

them.

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As long as educational research does not provide practical

ways of accomplishing this development, it is no doubt

right to give to the acquisition of symbolic logic a more

important share in the teaching process.

Inspiration may come from the evolution of symbolism in

mathematics through the centuries.

Howson alludes to this5 and the analogy of the evolution

of the individual’s knowledge according to Piaget’s theory.

Those noted in physics are arguments in the same direction;

if one begins with the hypothesis that human logic can

exist, it is likely that there are similarities.

But above all, so that the student finds his way naturally,

we should propose to him varied representations of the

same thing: “a supple and changing, suggestive and logical

formalism ” according to Lowenthal.9 We come back to

the recommendation of Howson and Brandsond: «no symbol

or contraction should be introduced if the student is not

ready fully and reasonably to appreciate the advantage it

offers».

We consider that the use of natural language along with

symbolic language can not only better guarantee the

acquisition of the symbolic language5,6 but above all serve

as a better basis or guide for the logical reasoning associated

with mathematical development.

*(v) Avoidable Difficulties*

As well as the intrinsic difficulties in the acquisition of the

symbolic language of mathematics, there exist difficulties

that one could avoid, growing out of the language used to

mediate between natural language and symbolic language.

This is the language used by teachers or school text to

give definitions, enunciate properties and theorems and to

provide the necessary explanations for beginners.

The language used by teachers is obviously very diverse

and varied, and there is no doubt that large numbers

of them know how to adapt as is necessary. In France the

General Inspectorate of Education encourages them to do

so. It recommends in particular that they avoid the introduction

of too many new words.

But if one considers school textbooks, one can ponder

whether these instructions have been taken into consideration.

The intellectual worth of the authors is not in question,

and one must seek the reason in an insufficient realisation

of the importance of the linguistic vehicle. We have

used a textbook for the level known as «5e» where, exceptionally,

a first chapter is devoted to helping in understanding

the terms used in the body of the text. So as to draw

a conclusion «a fortiori» we subjected this chapter to a test

for the «classification of texts according to the difficulty of

the approach required for understanding them» used in

technical education to select documents for students

according to their academic level.

This test has no pretentions to scientific perfection but

the results achieved demonstrate its pertinence.

The result is edifying: With respect to the French used,

this test should be given only to students three or four

years older. The analysis of difficulties shows essentially:

1. that the vocabulary used includes too many words

which are not part of the everyday language of the

student;

2. that certain known words are used in different

senses (paronyms);

3. that there is a supposition of certain references of

experience (not only mathematical but also of a

cultural nature);

4. that the structure of typical phrases aimed at

mathematical precision causes ambiguities on the

level of the French language.

As for the first two of these four observations, we carried

out a summary evaluation of the vocabulary requirements

of five of the most widely used textbooks. With respect to

the first level of the basic French vocabulary (representing

between 1,300 and 1,500 words) the comprehension of

the French used as a vehicle for mathematics teaching

(not including symbols) requires the knowledge of 100 to

150 new words or expressions.

In this body of material, the words that seem to be

known but which are used with a different meaning created

a doubly negative effect; they are not passive obstacles

to comprehension, but introduce confusion.

Several researchers1 0 have demonstrated this undesirable

effect of the most common of these: if (and only

if), then, and, or (exclusive), all ... These fundamental

words should be introduced with the same care as symbols

for they are not stepping stones to symbolic language,

they are merely its image.